account. Thus, dilute sprays are prone to ambient flow penetration, as expected.

In conclusion, the criterion presented herein is a sufficient condition for ambient flow to go around a given spray configuration, rather than penetrating it. Numerical calculations show that a spray will be more prone to ambient flow penetration for leaner mixtures, lower initial temperatures of gas phase and droplet, and more dilute spray configurations.

Similar calculations to those presented above based now upon a dynamic, rather than a static, criterion would greatly contribute to physical accuracy when imbedded in more complicated spray-in-turbulent-flow models. This is because in turbulent flows, eddies contain pockets of droplets that tend to have their own identity much as the cloud in this model. In order to properly model the evaporation of these pockets of drops, it is first necessary to determine the conditions to which they are exposed, in particular if they are exposed to a flow past them. The model developed above shows that such an estimate can be easily accomplished.

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On the relativistic temperature transformations and the related energy transport problem

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A LONG-STANDING difficulty extensively discussed in the literature pertains to the correctness of the transformation of temperature in special relativity [1, 2]. With the symbol T^r for the rest frame temperature (rest with respect to the center of mass of the system), the transformation is

$$T' = T^r\sqrt{1-v^2/c^2} \tag{1}$$

which is occasionally called the Einstein–Planck transformation although the alternative formula, equation (2), was also used by Planck [1]. Here v is the convection velocity of the system. Ott [3] and Arzelies [4] criticized the above expression and advocated the covariant transformation which is characterized by four-vector representations of heat and

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temperature. They obtain

$$T'' = T^r(\sqrt{1-v^2/c^2})^{-1}. \tag{2}$$

Landsberg [5] suggests an invariant transformation $T = T^r$ arguing that the temperature has no meaning since it cannot be measured in the rest frame. An extensive discussion related to the above problem can be found in the papers by Schmid [6], Pathria [7], Balazs [8] and others.

In considering the variational formulation of the heat conduction problems we have always used the proper frame temperature $T = \partial\rho_e^\circ/\partial\rho_s^\circ$ where ρ_e° and ρ_s° are the proper densities of energy and entropy, respectively. Such a temperature was a basic notion for the next considerations. However the proper frame we had to assume was that of 'moving with the entropy', this entropy being the quantity adjoint to the temperature T in the Gibbs equation. If the entropy flux is J_s and the entropy density is ρ_s then $v_s = J_s/\rho_s$ is the velocity associated with the entropy flow. It turns out that this velocity appears naturally in the formulae characterizing reversible and irreversible transfer of energy and entropy. For an equilibrium heat conductor at rest, $v = 0$ and of course $v_s = 0$. If however an irreversible heat transfer occurs it results in an entropy flux different from zero even in a resting conductor. The corresponding velocity is $v_s = J_s/\rho_s$ appears then as a consequence of the temperature gradient and it may be called 'entropy drift velocity'. The quantity v_s characterizes the entropy flow for both reversible and irreversible cases and it differs from zero for a resting nonequilibrium system which conducts the heat. The quantities J_s and ρ_s are well-defined components of the entropy four-vector and their measurement determines the velocity $v_s = J_s/\rho_s$. The thermal energy density ρ_e in the frame moving with the velocity v_s does not contain the flux contribution; it is therefore the rest internal energy ρ_e°. Therefore one has

$$\rho_e^{\text{proper}} = \rho_e^\circ(\rho_s^\circ) = \rho_e^\circ(\rho_s\sqrt{1-v_s^2/c^2}) \tag{3}$$

where c is the speed of light and ρ_s is the entropy density in the laboratory frame in which the observed entropy flux is J_s and the related velocity is v_s. From equation (3) the three various definitions of temperature can be formally derived. The proper (thermostatic) temperature is of course

$$T = \frac{\partial\rho_e^\circ}{\partial\rho_s^\circ}. \tag{4}$$

The partial derivative of the scalar ρ_e° with respect to the density ρ_s taken at the constant velocity v_s is

$$T_{v_s} = T\sqrt{1-v_s^2/c^2} = T\sqrt{1-J_s^2\rho_s^{-2}c^{-2}} \tag{5}$$

from equations (3) and (4).

This quantity resembles the Einstein–Planck definition of temperature, equation (1). There is an essential difference, however, namely in equation (1) v is the velocity of convection whereas in equation (5) v_s is the velocity of the frame moving with the transferred entropy, such that $v_s = J_s/\rho_s$.

It may be argued that the differentiation of the scalar ρ_e° with respect to the time component ρ_s of the four-vector J_s^α should be done keeping constant the remaining components of J_s^α rather than the velocity v_s.

Hence one may define

$$T_{J_s} = \left(\frac{\partial\rho_e^\circ}{\partial\rho_s}\right)_{J_s} = \frac{\partial(\rho_s\sqrt{1-J_s^2\rho_s^{-2}c^{-2}})|_{J_s}}{\partial\rho_s} = T/\sqrt{1-J_s^2\rho_s^{-2}c^{-2}} \tag{6}$$

or in terms of v_s

$$T_{J_s} = T/\sqrt{1-v_s^2/c^2}. \tag{7}$$

This corresponds to the Ott–Arzelies definition providing, however, that the velocity $v_s = J_s/\rho_s$ appears in T_{J_s} replacing the convection velocity v. The velocity v_s can vary along the 'entropy paths', i.e. along the streamlines assigned to the $J_s(x)$ field in a concrete heat conduction process. The virtue of using temperatures T_{v_s} and T_{J_s} is that they do not contain the convection velocity explicitly. When a nonequilibrium conductor is in motion the observed flux J_s and density ρ_s differ from those measured for the resting conductor. They can be computed using well-known formulae for the transformation of the components of any four vector. Equations (4)–(7) are unchanged but in the general case involving convection, J_s is the *total* entropy flux containing the convective contribution. The definition of T in the frame 'moving with the entropy' is just as natural as that for the chemical potential μ of particles in the frame which moves with them, e.g. for electrons drifting in a resting nonequilibrium conductor, the natural definition of μ is in the frame of vanishing current. This approach sorts out the kinetic energy of electron drift which is, in fact, the essential quantity influencing the form of the electron transport equation. The distinction between the two frames, the proper frame of the conductor and the proper frame of the transported particles, becomes more and more essential when the diffusion fluxes increase. When the transfer of entropy or heat is considered it appears that the choice of T', T'', T_{v_s}, T_{J_s} is not a problem of definition but rather of the physical analysis. To reveal which expression should be favored let us consider the pure transfer of energy when the energy carriers are the massless particles.

Consider the action based on the proper frame internal energy density ρ_e° which is the usual Lagrangian density of a reversible process. For our problem of the irreversible transfer of energy the action should contain the additional multiplicative term containing the energy relaxation time τ in the form $\exp(t\tau^{-1})$. Such terms appear naturally in the problems of irreversible diffusion of heat and mass [9–11]. They are the scattering terms which describe interaction of a diffusive quantity with the conductor. Consequently we investigate the action:

$$\tilde{S} = -\iiiint \exp(t\tau^{-1})\rho_e^\circ(\rho_s\sqrt{1-J_s^2c^{-2}\rho_s^{-2}})\,\mathrm{d}V\,\mathrm{d}t \tag{8}$$

extremized under the constraint of the entropy balance equation

$$\frac{\partial\rho_s}{\partial t} + \nabla\cdot J_s = \sigma_s(x, t). \tag{9}$$

Here $\sigma_s(x, t)$ is the entropy source, as an explicit function of coordinates and time. Applying the method of Lagrangian multipliers, we consider the variation of the augmented functional

$$\tilde{S} = \iiiint \left\{ -\rho_e^\circ(\rho_s\sqrt{1-J_s^2c^{-2}\rho_s^{-2}}) + \varphi\left[\frac{\partial\rho_s}{\partial t} + \nabla\cdot J_s - \sigma_s(x, t)\right]\right\} \exp(t\tau^{-1})\,\mathrm{d}V\,\mathrm{d}t \tag{10}$$

where φ is a Lagrangian multiplier. The variation of $\tilde{S}$ is

$$\delta\tilde{S} = \iiiint \exp(t\tau^{-1})\left[\frac{Tv_s}{c^2\sqrt{(1-v_s^2/c^2)}}\,\delta J_s - \frac{T}{\sqrt{(1-v_s^2/c^2)}}\,\delta\rho_s + \varphi\left(\frac{\partial\delta\rho_s}{\partial t} + \nabla\cdot\delta J_s\right)\right]\mathrm{d}V\,\mathrm{d}t. \tag{11}$$

The condition $\delta\tilde{S} = 0$ leads to the equations

$$\frac{Tv_s}{c^2\sqrt{(1-v_s^2/c^2)}} - \nabla\varphi = 0 \tag{12}$$

and

$$\frac{T}{\sqrt{(1-v_s^2/c^2)}} + \frac{\partial\varphi}{\partial t} + \frac{\varphi}{\tau} = 0. \tag{13}$$

They are the stationarity conditions of $\tilde{S}$ with respect to J_s and ρ_s, respectively. At this point no conclusion can be made about the preference of the temperature (5) or (7). Although the temperature T_{J_s}, rather than T_{v_s}, appears in equations (12) and

(13) this is only the formal consequence of the variables used (J_s, ρ_s), with respect to which the action $\tilde{S}$ is varied. If the variables were (v_s, ρ_s) the temperature T_{v_s} would appear in the stationarity conditions.

Consider, however, the equation of motion implied by the conditions (12) and (13). The multiplier φ is easily eliminated by differentiating equation (12) with respect to t and equation (13) with respect to x and then adding equations so obtained, to get

$$\frac{1}{c^2}\frac{\partial}{\partial t}\left(\frac{Tv_s}{\sqrt{(1-v_s^2/c^2)}}\right)+\nabla\left(\frac{T}{\sqrt{(1-v_s^2/c^2)}}\right)+\tau^{-1}\frac{Tv_s}{c^2\sqrt{(1-v_s^2/c^2)}}=0. \quad (14)$$

This result is independent of the variables with respect to which the action $\tilde{S}$ was varied. It suggests that the temperatures T_{J_s} or T should be preferred, as they are the quantities in terms of which the equation of thermal motion (14) can be written in the simplest possible manner. Since $v_s = J_s/\rho_s$, equation (14) can be transformed into the form

$$\tau\rho_s\frac{\partial}{\partial t}(\rho_s^{-1}J_e)+J_e=-\rho_s D\nabla T_{J_s}=-\lambda_e\nabla T_{J_s} \quad (15)$$

where $D = \tau c^2$ is the coefficient of diffusivity, $\lambda_e = \rho_s D$ and J_e is the energy flux defined (since there is no mass transfer) by

$$J_e \equiv T_{J_s}J_s=\frac{T}{\sqrt{(1-v_s^2/c^2)}}J_s. \quad (16)$$

Equation (15) describes the energy diffusion when the energy carrier is the massless relativistic particle. It has the structure of an extended Fourier–Fick law with the diffusivity $D = \tau c^2$ as the product of a single relaxation time and the square of the propagation velocity c. (This is in accordance with the usual definition of diffusivity for mass transfer which appears in Fick's law.) The product $\rho_s D = \lambda_e$ plays the role of the energy conductivity.

Equation (15) is strongly nonlinear because the entropy density (and hence λ_e) depends on the temperature T. The presence of an unsteady-state term in (15) indicates that energy flux exhibits inertia even if its carrier is a particle with zero rest mass, consistent with momentum transport by such particles. In fact (15) has the structure of Maxwell's [12] generalization of the Fourier–Fick law of diffusion.

The definition of energy flux (16) seems to be somewhat arbitrary up to this point. We will now show that the product $T_{J_s}J_s$ is the proper physical quantity since it is precisely the quantity which appears in the energy balance expression. Computing the time derivative of the energy density $\partial\rho_e^\circ/\partial t$ as

$$\frac{\partial\rho_e^\circ}{\partial t}=-\frac{Tv_s}{c^2\sqrt{(1-v_s^2/c^2)}}\frac{\partial J_s}{\partial t}+\frac{T}{\sqrt{(1-v_s^2/c^2)}}\frac{\partial\rho_s}{\partial t} \quad (17)$$

one can eliminate $\partial\rho_s/\partial t$ from the above equation using the balance equation (9). The result can be transformed into the form

$$\frac{\partial\rho_e^\circ}{\partial t}+\mathrm{div}\left(\frac{T}{\sqrt{(1-v_s^2/c^2)}}J_s\right)=-\frac{\partial}{\partial t}\left(\frac{T}{\sqrt{(1-v_s^2/c^2)}}\frac{J_s^2}{\rho_s c^2}\right)+T\sigma_s+J_s\left(\nabla\frac{T}{\sqrt{(1-v_s^2/c^2)}}+\frac{\partial}{\partial t}\frac{Tv_s}{c^2\sqrt{(1-v_s^2/c^2)}}\right). \quad (18)$$

But the last term of this equation can be simplified by using the equation of motion (14). Finally we obtain ($\lambda_e = \rho_s D$)

$$\frac{\partial\rho_e^\circ}{\partial t}+\mathrm{div}\left(\frac{TJ_s}{\sqrt{(1-v_s^2/c^2)}}\right)=-\frac{\partial}{\partial t}\left(\frac{TJ_s}{\sqrt{(1-v_s^2/c^2)}}\frac{J_s}{\rho_s c^2}\right)+T\sigma_s-\left(\frac{T}{\sqrt{(1-v_s^2/c^2)}}\right)^{-1}\lambda_e^{-1}\left(\frac{TJ_s}{\sqrt{(1-v_s^2/c^2)}}\right)^2 \quad (19)$$

where the last term on the RHS of equation (19) describes the negative contribution to the energy balance due to dissipation. The nonrelativistic counterpart of (19) has the well-known form of the energy balance

$$\frac{\partial\rho_e}{\partial t}+\mathrm{div}\,J_e=T\sigma_s-T^{-1}\lambda_e^{-1}J_e^2 \quad (J_e=TJ_s). \quad (20)$$

The last terms in equations (19) and (20) describe irreversible energy dissipation. The correspondence of equation (19) with (20) is not only formal since, in the case of no mass transfer the LHS of (19) is precisely the four-divergence of the energy momentum tensor. Therefore the temperatures T_{J_s} and T are the natural physical quantities and (16) describes the natural definition of the energy flux. The temperature T_{J_s} appeared naturally also in more complicated transfer models we have considered, involving energy transport by collision of the nonzero mass particles [13].

Although the temperature T_{J_s} appears commonly in the energy transport formulas and in the energy momentum tensor it is always expressed in terms of the most fundamental quantity, the rest temperature T measured in the frame of vanishing entropy flux. The temperature T constituted, in fact, the starting quantity for all derivations given here. When a complex high-flux transport process occurs with various diffusing species present, it may be necessary to define a number of the quantities analogous to T. They will be related to the partial entropies of the species and the problem of multiple temperature relaxation will arise.

In conclusion the definition of the relativistic transformation of temperature can be linked with the pure energy transfer via the relativistic diffusion in a nonequilibrium system. The corresponding equation of diffusion has the form of Maxwell's generalization of the Fourier–Fick law. In this work, only the case of massless energy carriers has been considered. The energy transfer equation having the extended structure of equation (15) can be derived also when the energy transport occurs via the diffusion of particles with nonzero rest mass, e.g. molecules of a perfect gas. This is, in fact, a classical problem of heat conduction in a gas discussed by Feynman *et al.* [14]. Therefore it should not be concluded that the relaxation structure of the energy transport equation or the inertial property of energy flux have relativistic origin.

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Axial heat conduction effects in natural convection along a vertical cylinder

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INTRODUCTION

Many transport processes occur in nature and in industrial applications in which the transfer of heat is governed by the process of natural convection. Natural convection arises in fluids when the temperature changes cause density variations leading to buoyancy forces. An excellent review of natural convection flows has been given by Ede [1]. Recently, Minkowycz and Sparrow [2, 3], Cebeci [4], and Aziz and Na [5] have studied the steady, laminar, incompressible, natural convection flow over a vertical cylinder using a local nonsimilarity method, a finite-difference scheme, and an improved perturbation method, respectively. However, they did not take into account the effect of axial heat conduction for small Prandtl number. It is known that the axial heat conduction effect becomes important for low-Prandtl-number fluids such as a liquid metal.

The aim of the present analysis is to study the effect of axial heat conduction on the steady, laminar, incompressible, natural convection flow over a vertical cylinder. The partial differential equations governing the flow have been solved numerically using an implicit finite-difference scheme in combination with the quasilinearization technique [6]. The results have been compared with the available results [2–5].

GOVERNING EQUATIONS

We consider a thin, vertical cylinder of radius r_0 which is situated in a quiescent environment having temperature T_∞. The surface of the cylinder is maintained at a uniform temperature T_w. The axial and radial coordinates are taken to be x and r, with x measuring the distance along the centerline of the cylinder from its bottom end and r measuring normal to the axis of the cylinder. The gravitational force acts in the opposite direction to x. The fluid is assumed to have constant

NOMENCLATURE

F, F_w	dimensionless streamfunction and mass transfer parameter, respectively
F''_w, G'_w	skin friction and heat transfer parameters, respectively
g, G	gravitational acceleration and dimensionless temperature, respectively
Gr, Gr_x	Grashof number and local Grashof number, respectively
Nu, Pr	Nusselt number and Prandtl number, respectively
r, x	radial and axial coordinates, respectively
r_0	radius of cylinder
T	temperature
u, v	velocity components in x- and r-directions, respectively.

Greek symbols

α, β	thermal diffusivity of the fluid and volumetric coefficient of thermal expansion, respectively
ξ, η	transformed coordinates
λ, ν, ψ	axial heat conduction parameter, kinematic viscosity, and dimensional streamfunction, respectively.

Superscript

$'$	differentiation with respect to η.

Subscripts

x, r, ξ	derivatives with respect to x, r and ξ, respectively
w, ∞	conditions at the wall and in the free stream, respectively.

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